Derivatives

1. Show that the function $y = c_1e^{2x}+c_2xe^{2x}+e^x$ satisfies the differential equation $y''-4y'+4y = e^x$.

2. Show that the function $y = e^{-x}\sin x$ satisfies the differential equation $y'' + 2y' + 2y = 0$.

3. Calculate the derivative of the function:

$$y = -\frac{\cos x}{\sin^2 x} + \ln \operatorname{tg} \frac{x}{2}.$$

Equation of the Tangent to a Graph

4. Find the equation of the tangent and normal to the parabola $y = 4 - x^2$ at the point where $x = -1$.

5. Find the equation of the tangent to the graph $y = \operatorname{arc}\sin x$ at the point $x = \frac{1}{2}$.

6. Find the intersection of the tangents to the curve $y = \frac{1}{1+x}$ at the points $x = -3$ and $x = -\frac{1}{2}$.

7. Prove that the length of the tangent segment between the coordinate axes for an astroid is constant! The astroid is given by the equation $x^{2/3} + y^{2/3} = \alpha^{2/3}$, where α is a positive number.

8. Find the normal to the graph of the real function $f(x) = x^2 + 3x - 1$ that intersects it at the point with the abscissa $x = 0$.

Linear Approximation

9. Using the differential (linear approximation), approximate $\sqrt{1.05}$.

10. Using the differential, approximate the value of $\ln(0.95)$.

11. Using the differential (linear approximation), approximate $\sqrt[3]{8.05}$.

12. Using the differential (linear approximation), approximate $\cos\left(\frac{2\pi}{9}\right)$.

13. Using the differential (linear approximation), approximate $\sqrt{21}$. Solve the problem by considering the function $f(x) = \sqrt{10x - x^2}$ at the point $x_0 = 2$.

All above math problems are taken from the following website:
`https://osebje.famnit.upr.si/~penjic/teaching.html`.
The reader can find all solutions to the given problems on the same page.